J. Fluid Mech. (1972), *vol.* 55, *part* 3, *pp.* 473–492
Printed in Great Britain

An investigation of high-wavenumber temperature and velocity spectra in air

By NOEL E. J. BOSTON† AND R. W. BURLING
Institute of Oceanography, University of British Columbia, Vancouver, Canada

(Received 9 June 1971 and in revised form 6 July 1972)

Turbulent temperature and velocity fluctuations in air were measured at a height of 4 m over a tidal mud flat. Particular attention was focused on the high-wavenumber, small-scale region of the spectra of these fluctuations. The measurements of the velocity fluctuations were made with a constant-temperature hot-wire anemometer; the hot wire consisted of a platinum wire 5 μm in diameter and approximately 1 mm in length. Temperature fluctuations were measured with a platinum resistance thermometer which consisted of a platinum wire 0·25 μm in diameter and about 0·30 mm in length.

The velocity spectra results agree well with the classical results of Grant, Stewart & Moilliet (1962) and Pond, Stewart & Burling (1963). In addition, they extend the velocity spectrum in air to slightly higher wavenumbers. The one-dimensional Kolmogorov constant K' estimated from these data was 0·51.

The temperature spectra clearly show the shape of the one-dimensional temperature spectrum in air beyond the $-\frac{5}{3}$ region. In air temperature and velocity spectra are very similar. The value of the scalar constant K_θ', which appears in the scalar $-\frac{5}{3}$ law, computed from these data was 0·81. Direct measurement was made of all parameters that enter into the calculation of it.

1. Introduction

During the past ten years attempts have been made to measure the full one-dimensional spectrum of temperature fluctuations in the atmospheric boundary layer. The distribution of temperature in a turbulent field has a direct bearing on a number of problems in and related to geophysics, since resulting inhomogeneities influence the scattering of sound waves and electromagnetic radiation. The important parameter in such scattering problems is the refractive index, the variation of which is often related to the small-scale structure of the temperature distribution. For waves in the optical and infra-red regions, fluctuations in the refractive index are determined primarily by density fluctuations which in turn can be related to temperature fluctuations. In the case of laser propagation, even the smallest scales of these fluctuations can be significant. Further application of a knowledge of the small-scale distribution of temperature is found in the design of such engineering systems as chemical reactors, the analysis of the mixing

† Present address: Department of Oceanography, Naval Postgraduate School, Monterey, California 93940.

of two fluids of equal density, and heat transfer. The subject of heat transfer is especially significant to studies in oceanography and meteorology. Besides these applications, high-wavenumber spectra are of great interest in their own right, since they lead to improved understanding of turbulence in general both as a source of new ideas and as a testing field for theory.

2. Theoretical relations

2.1. *Velocity*

Theoretical discussions usually use the three-dimensional energy (velocity) spectral density function $E(k)$ (Batchelor 1953, p. 36), since this function seems to come closest to the physical concept of the energy per unit mass associated with a particular scale of motion. It may be defined so that

$$\int_0^\infty E(k)\,dk = \tfrac{1}{2}(\overline{u_1^2}+\overline{u_2^2}+\overline{u_3^2}), \tag{1}$$

where the wavenumber k is the amplitude of the vector $\mathbf{k}$, and u_1, u_2, and u_3 are Cartesian turbulent velocity components. The bar denotes an ensemble average which is equivalent to a space or time average under certain assumptions (Lumley & Panofsky 1964, p. 6 ff.) which are satisfied provided the turbulence field is reasonably 'stationary'. In isotropic turbulence these velocities are the total velocities, since any mean motion may be removed by a simple co-ordinate transformation which does not change the statistical properties of the turbulence. In anistropic turbulence these velocities are the deviations from the mean. Ensemble averages are used, at least in principle, since they allow the interchange of averaging and differentiation.

Because of the difficulties of measuring $E(k)$ directly, experimentally, the one-dimensional spectrum $\phi(k)$ is measured. It is defined so that

$$\int_0^\infty \phi(k_1)\,dk_1 = \overline{u_1^2}, \tag{2}$$

where k_1 in this integral represents the component of $\mathbf{k}$ in the X_1 direction. For isotropic turbulence $\phi(k_1)$ is related to $E(k)$ by (Hinze 1959, p. 171)

$$2E(k_1) = k_1^2\frac{\partial^2\phi(k_1)}{\partial k_1^2} - k_1\frac{\partial\phi(k_1)}{\partial k_1}. \tag{3}$$

Also, in isotropic turbulence the average rate of energy dissipation per unit mass is given by (Hinze 1959)

$$\epsilon = 2\nu\int_0^\infty k^2E(k)\,dk = 15\nu\int_0^\infty k_1^2\phi(k_1)\,dk_1 = 15\nu\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}, \tag{4}$$

where ν is kinematic viscosity, $k^2E(k)$ and $k_1^2\phi(k_1)$ are referred to as dissipation spectra and describe the distribution in wavenumber space of the average rate of decay of turbulent energy to heat. From (4), it is evident that $k_1^2\phi(k_1)$ is the spectrum of $\partial u_1/\partial x_1$. Relations (4) will be valid in turbulence which is only locally isotropic, provided that the major contributions to the integrals are from wavenumbers which are locally isotropic.

An important idea developed in modern turbulence theory is that of the energy cascade: large-scale motions are being constantly degraded into smaller and smaller scales. Kolmogorov (1941) applied the idea of the energy cascade seriously and developed a theory that should be applicable to all fields of turbulence provided the Reynolds number (*Re*) is high enough (i.e. the ratio of inertial to viscous forces is large). A high *Re* guarantees that the wavenumber separation between the energy-containing region of the spectrum and the viscous dissipation region is large. The small-scale turbulence has lost its identity with the large-scale generating forces and eventually becomes isotropic (Taylor 1938).

The first Kolmogorov hypothesis states that for sufficiently high *Re*, statistical properties of locally isotropic scales are uniquely determined by ϵ and $\nu[\mathrm{cm}^2\,\mathrm{s}^{-1}]$. Dimensional analysis then yields

$$\phi(k_1) = (\epsilon\nu^5)^{\frac{1}{4}} F(k_1/k_s) \tag{5}$$

for the form of the one-dimensional energy spectrum and

$$k_1^2\phi(k_1) = k_s^2(\epsilon\nu^5)^{\frac{1}{4}} (k_1/k_s)^2 F(k_1/k_s) \tag{6}$$

for the form of the energy dissipation spectrum. In these relations $k_s (= (\epsilon/\nu^3)^{\frac{1}{4}})$ is the Kolmogorov wavenumber, its reciprocal being referred to as the Kolmogorov microscale. Provided *Re* is high enough, $F(k_1/k_s)$ should be a universal function valid for all fields of turbulence.

Universal similarity of velocity spectra for water and air for grid and oceanic turbulence was first demonstrated by Gibson (1962) and Gibson & Schwarz (1963). They used wind tunnel data from Stewart & Townsend (1951), their own water tunnel data, and compared the results with the independent, simultaneous normalized spectra which Grant *et al.* (1962) had obtained from measurements in a tidal channel. The results of Pond *et al.* (1963) from atmospheric measurements, offered additional striking support.

The second Kolmogorov hypothesis states that if *Re* is sufficiently large, there should exist a range of scales which are isotropic and in a local steady state but for which viscosity is not important. The statistical properties of the turbulence are determined by ϵ alone. Dimensional analysis then leads to the now well-known law of local isotropy,

$$\phi(k_1) = K'\epsilon^{\frac{2}{3}} k_1^{-\frac{5}{3}}, \tag{7}$$

where K' is a universal constant. From equation (3) it follows that, if the one-dimensional spectrum has a negative power law form, then the three-dimensional spectrum has the same power law behaviour. In the inertial range then, from (3) and (7),

$$E(k) = K\epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}, \quad K = \tfrac{55}{18}K'. \tag{8}$$

The constants K and K' also are referred to as α and α_1, respectively in other literature. The results of Grant *et al.* and of Pond *et al.* also confirm (7), as their data exhibit a long region where the slope is $-\frac{5}{3}$ on a log–log plot. In fact their $-\frac{5}{3}$ region extended to anomalously low wavenumbers, where the turbulence could not possibly be isotropic. These and subsequent measurements (e.g. Weiler & Burling 1967) have led to a re-evaluation of ideas on local isotropy. It is interesting to note that, in the early 1960's, turbulence experiments tended to confirm

Kolmogorov's classical similarity theory. At the same time Kolmogorov (1962) revised his theory to take into account the variability of ϵ, an omission which had been pointed out by Landau shortly after the development of the original similarity theories. Since 1962, and especially recently (Gibson, Stegen & Williams 1970; Stewart, Wilson & Burling 1970; Wyngaard & Tennekes 1970; Sheih, Tennekes & Lumley 1971), evidence has accumulated favouring Kolmogorov's 1962 refinement. In particular, the refined theory and current evidence suggest that the velocity spectral shape is not universal, but depends on turbulence Reynolds number. The previous discussion should be viewed in light of these recent developments.

According to the original Kolmogorov ideas, the constant K' of equation (7) should be a universal one, independent of the particular fluid or of the mean flow and the spectrum at low wavenumbers. Pond *et al.* (1966) summarized values of K' for four different flow fields (atmosphere (Pond), laboratory air jet (M.M. Gibson 1963), ocean (Grant *et al.* 1962) and water tunnel (Gibson & Schwarz 1963)). The mean value of K' was 0·48. Since the Pond *et al.* (1966) paper, other experimental results have been published. Kistler & Vrebalovich (1966) reported values of K' of 0·65 in very high Reynolds number grid turbulence. Shieh *et al.* (1971) obtained a value of 0·65 in atmospheric measurements at high Reynolds number and Gibson *et al.* (1970) obtained a value of 0·69 from atmospheric measurements over the Atlantic Ocean. In addition, two theoretical values are available. Kraichnan (1968) derived $K' = 0{\cdot}58$ using the abridged Lagrangian history direct interaction approximation, and Pao (1965) obtained a value of 0·55 by fitting his spectral cut-off function to inertial subrange measurements with K' as the adjustable parameter. To this list of K' values may be added the values of 0·53 obtained by Stewart *et al.* (1970) from measurements of the skewness of velocity derivatives, 0·56 obtained by Nasmyth (1970) from a new universal curve based on a very clean oceanic turbulence data, and 0·57 obtained by Paquin & Pond (1971) from second- and third-order structure functions computed from velocity fluctuations in the wind over the ocean.

According to the new Kolmogorov ideas, K' may not be a universal constant but a function of the macrostructure of the flow. There presently exists the choice of a K' which is Reynolds number dependent (Wyngaard & Tennekes 1970) or a constant K' with a steeper slope in the inertial subrange (Yaglom 1966). Regardless, the Re dependence is weak and difficult to detect experimentally.

These recent results cast doubt on the idea that the value of K' is exactly 0·48. This paper adds further data on K', which tend to support the original estimates.

2.2. *Temperature*

Kolmogorov's ideas, both old and new, may be applied to turbulent fluctuations of passive scalars such as temperature fluctuations in the atmosphere and oceans, salt concentration fluctuations in the oceans or dye concentration fluctuations in laboratory experiments. A passive scalar is any quantity (including contaminants) that modifies and/or can be transported in a fluid but which does not introduce buoyancy effects. A scalar which did introduce buoyant type forces would be considered to be active. 'Temperature' is active at large generating scales but not at the small scales discussed here. The symbol 'θ' will be used to indicate 'intensity' of a scalar, or as a subscript to indicate quantities which refer to scalar quantities.

If θ is the deviation of a scalar property from its 'ensemble' mean, the spectral functions for its fluctuations are defined so that

$$\overline{\theta^2} = \int_0^\infty E_\theta(k)\,dk, \quad \overline{\theta^2} = \int_0^\infty \phi_\theta(k_1)\,dk_1, \tag{9), (10}$$

where $E_\theta(k)$ is the three-dimensional spectrum, and $\phi_\theta(k_1)$ is the one-dimensional spectrum. These spectra are related in the isotropic case (Hinze 1959, p. 226) by

$$E_\theta(k_1) = -k_1\frac{\partial\phi_\theta(k_1)}{\partial k_1}. \tag{11}$$

The mean rate of scalar dissipation ϵ_θ is determined by

$$\epsilon_\theta = 2\kappa\int_0^\infty k^2E_\theta(k)\,dk = 6\kappa\int_0^\infty k_1^2\phi_\theta(k_1)\,dk_1, \tag{12}$$

provided that the turbulence is isotropic at least in the range of wavenumbers which provide the dominant contribution to the integrals. κ is the diffusivity of the scalar property. $k^2E_\theta(k)$ and $k_1^2\phi_\theta(k_1)$ are scalar dissipation spectra and describe the distribution with wavenumber of the rate of decay of scalar fluctuations. The second expression in (12) merits further discussion. An alternative expression for ϵ_θ uses a 3κ in place of 6κ in front of the integral. The difference originates in the form of the scalar dissipation budget used. For the budget of $\overline{\theta^2}$, 6κ is needed, whereas in the budget of $\overline{\tfrac{1}{2}\theta^2}$, 3κ is needed. A discussion of this problem is presented by Paquin & Pond (1971, p. 267).

Gibson (1968*b*) introduced a set of similarity hypotheses for locally isotropic scalar fields. Since his notation is slightly different from that used here, the hypotheses will be reworded.

(i) At sufficiently high *Re* the statistical properties of the scalar fluctuations are determined by the mean rate of strain $\gamma[=(\nu/\epsilon)^{\frac{1}{2}}]$, ϵ_θ and κ. Dimensional analysis then leads to

$$\phi_\theta(k_1) = \epsilon_\theta\epsilon^{-\frac{3}{4}}\nu^{\frac{5}{4}}H(\sigma, k_1/k_s) \tag{13}$$

for the one-dimensional scalar spectrum and

$$k_1^2\phi_\theta(k_1) = k_s^2\epsilon_\theta\epsilon^{-\frac{3}{4}}\nu^{\frac{5}{4}}(k_1/k_s)^2H(\sigma, k_1/k_s) \tag{14}$$

for the scalar dissipation spectrum. In these relations σ is the Prandtl number ($\sigma = \nu/\kappa$), the ratio of kinematic viscosity to molecular diffusivity.

(ii) If there exists a range of wavenumbers which is isotropic, but in which neither viscosity nor diffusivity is important, then the only determining parameters are ϵ_θ and ϵ. Dimensional arguments then lead to

$$\phi_\theta(k_1) = K'_\theta\epsilon_\theta\epsilon^{-\frac{1}{3}}k_1^{-\frac{5}{3}}, \tag{15}$$

where K'_θ is a universal constant.

These hypotheses are covered by Gibson's similarity hypotheses (1) (1968*b*), p. 2318) for scalar fields of arbitrary diffusivity. His equation (32) is equivalent to (13) and his equation (33) is the same as (15).

Equations (15) and (17) were first tested for ϵ_θ, ϵ, k_1 and σ dependence by Gibson & Schwarz (1963) for the cases of temperature and salt concentration fluctuations in water, in a water tunnel. Gurvich & Kravchenko (1962) and Pond (1965) verified the k_1 dependence of (15) for temperature fluctuations in air.

From (11) it follows that, if the one-dimensional scalar spectrum has a negative power law form, then the three-dimensional spectrum has the same power law behaviour. For the isotropic range of wavenumbers, from (11) and (15),

$$E_\theta(k) = K_\theta \epsilon_\theta \epsilon^{-\frac{1}{3}} k^{-\frac{5}{3}}, \quad K_\theta = \tfrac{5}{3} K'_\theta. \tag{16}$$

Whereas the $-\frac{5}{3}$ form of the temperature fluctuation spectrum has been demonstrated, the form at scales where viscosity and conductivity are important is not clear. Theoretical studies have enjoyed only partial success. Obukhov (1949) and Corrsin (1951) estimated the drop from the $-\frac{5}{3}$ power law to occur at $k_c = (\epsilon/\kappa^3)^{\frac{1}{4}}$. Batchelor (1959) argued that this cut-off wavenumber occurs only when $\sigma \leqslant 1$. For $\sigma \gg 1$, Batchelor suggested the existence of a viscous–convective subrange $[(\epsilon/\nu\kappa^2)^{\frac{1}{4}} \ll k \ll (\epsilon/\nu\kappa^2)^{\frac{1}{4}}]$ and a viscous–diffusive subrange $[(\epsilon/\nu\kappa^2)^{\frac{1}{4}} \ll k]$. He proposed a uniform straining model for these ranges and obtained the spectrum function

$$E_\theta(k) = -\epsilon_\theta \gamma^{-1} k^{-1} \exp[\kappa k^2/\gamma] \quad (k \ll k_s), \tag{17}$$

where $\gamma = -(\frac{1}{2})(\epsilon/\nu)^{\frac{1}{2}}$. For the case of strongly diffusive scalars ($\sigma \ll 1$), Batchelor, Howells & Townsend (1959) proposed the existence of an inertial–diffusive subrange and obtained

$$E_\theta(k) \simeq (\tfrac{1}{3}) \epsilon_\theta \epsilon^{\frac{2}{3}} \kappa^{-3} k^{-\frac{17}{3}} \quad (k_c \ll k \ll k_s). \tag{18}$$

Gibson (1968*a*) stressed the importance of the local rate of strain to the development of the largest wavenumber perturbations of the scalar field, and in the companion paper mentioned already (Gibson 1968*b*) developed spectral predictions appropriate to various ranges of scale and Prandtl numbers. For the case of small σ he obtained a k^{-3} dependence for the scalar spectrum in the range $k_c \ll k \ll k_s$.

Some experimental verification of (17) has been obtained by Grant *et al.* (1968), who measured temperature and velocity fluctuations in the ocean. Experimental work in low σ fluids (Rust & Sesonke 1966; Granatstein, Buchsbaum & Bugnolo 1966) have yielded results that do not unambiguously support a specific theory, although it is interesting to note that Rust & Sesonke put a -3 line through their spectra without any knowledge of either the Batchelor *et al.* (1959) or the Gibson (1968*b*) theory. For the intermediate case (i.e. σ not large or small) no clear definitive theory is available, though the problem has been considered by Howells (1960), Pao (1965), Kraichnan (1968) and Gibson (1968*a*, *b*). For unity Prandtl number fluids, Van Atta (1971) has derived inertial subrange expressions which take into account the fluctuations in dissipation rates of scalar and velocity fields. He obtained a wavenumber dependence in the inertial subrange which varies about the traditional $-\frac{5}{3}$ (or $-1{\cdot}67$) power law form, namely $-1{\cdot}56$ to $-1{\cdot}72$. The intermediate Prandtl number case is extremely important, since it probably applies to air and other gases (for which

$\sigma \sim 1$). The modifier 'probably' is used, because it is not at all clear whether a Prandtl number of unity should be considered large or small or neither.

The constant K_θ' of equation (15), according to original Kolmogorov ideas, is expected to be universal. Since scalar spectra at high wavenumbers are not nearly as well documented as velocity spectra, the value of K_θ' is in some doubt, doubt that has been compounded by definitions of spectra and of (12) which differ by a factor of two. The first measurement of the constant was made by Gibson & Schwarz (1963), who obtained $K_\theta' = 0{\cdot}35$. Since then a variety of values has been reported. Experimentally these range from 0·31 (Grant *et al.* 1968) to 2 (Gurvich & Zubkovsky 1966) and theoretically from 0·208 (Kraichnan 1968) to 0·9 (Gibson 1968*b*). This paper presents additional values of K_θ' for the case of temperature fluctuations in air.

3. Experiment

The measurements were made at Boundary Bay, British Columbia (figure 1). All instruments were supported from a portable mast at a height of 4 m. The signals from the sensors were carried by cables to a panel truck which served as the housing for the recording equipment. The truck was 30 m downwind from the mast.

Turbulent velocity was measured with a constant-temperature hot-wire anemometer system (DISA Model 55D05). The unit was battery operated and used in a 1:1 bridge ratio mode which requires a compensating cable in the balance arm whose impedance matches that of the probe cable. The hot-wire probe consisted of a platinum wire $5\,\mu$m in diameter and 1·0 mm in length. It was mounted approximately 5 cm above the temperature sensor.

Mean wind speed was monitored by a cup anemometer (Thornthwaite Associates).

The sensor of temperature fluctuations was a resistance thermometer consisting of a platinum wire $0{\cdot}25\,\mu$m (0·00001 in.) in diameter and about 0·30 mm in length (figure 2, plate 1). Direct measurements of the time constant of the wire indicated that its frequency response was flat to beyond 2 kHz. Calculations based on empirical formulae supported this result. Assuming Taylor's hypothesis in the form $k = 2\pi f/U$ and $U = 600\,\mathrm{cm}^{-1}\,\mathrm{s}$ as the maximum wind speed to be encountered (for this experiment), $f = 1$ kHz as the highest frequency of interest, then the maximum wavenumber of interest is $k \simeq 10\,\mathrm{cm}^{-1}$. Since the wavenumber associated with the length of the wire is in excess of $30\,\mathrm{cm}^{-1}$, then the spatial resolution of this wire is more than adequate. The dimensions of the wire also ensured that end effects (due to a length to diameter ratio of approximately 1200) and other effects would be negligible.

The electronic system used to convert resistance changes of the wire to usable voltage fluctuations consisted of an 80 kHz multivibrator, bridge, bridge amplifier, synchronous detector and d.c. amplifier (figure 3). Its frequency response is flat from d.c. to approximately 10 kHz. Further circuit details can be found elsewhere (Boston 1970). In order to be certain that no significant velocity dependence appeared in the temperature signal, the $0{\cdot}25\,\mu$m diameter wire was operated with

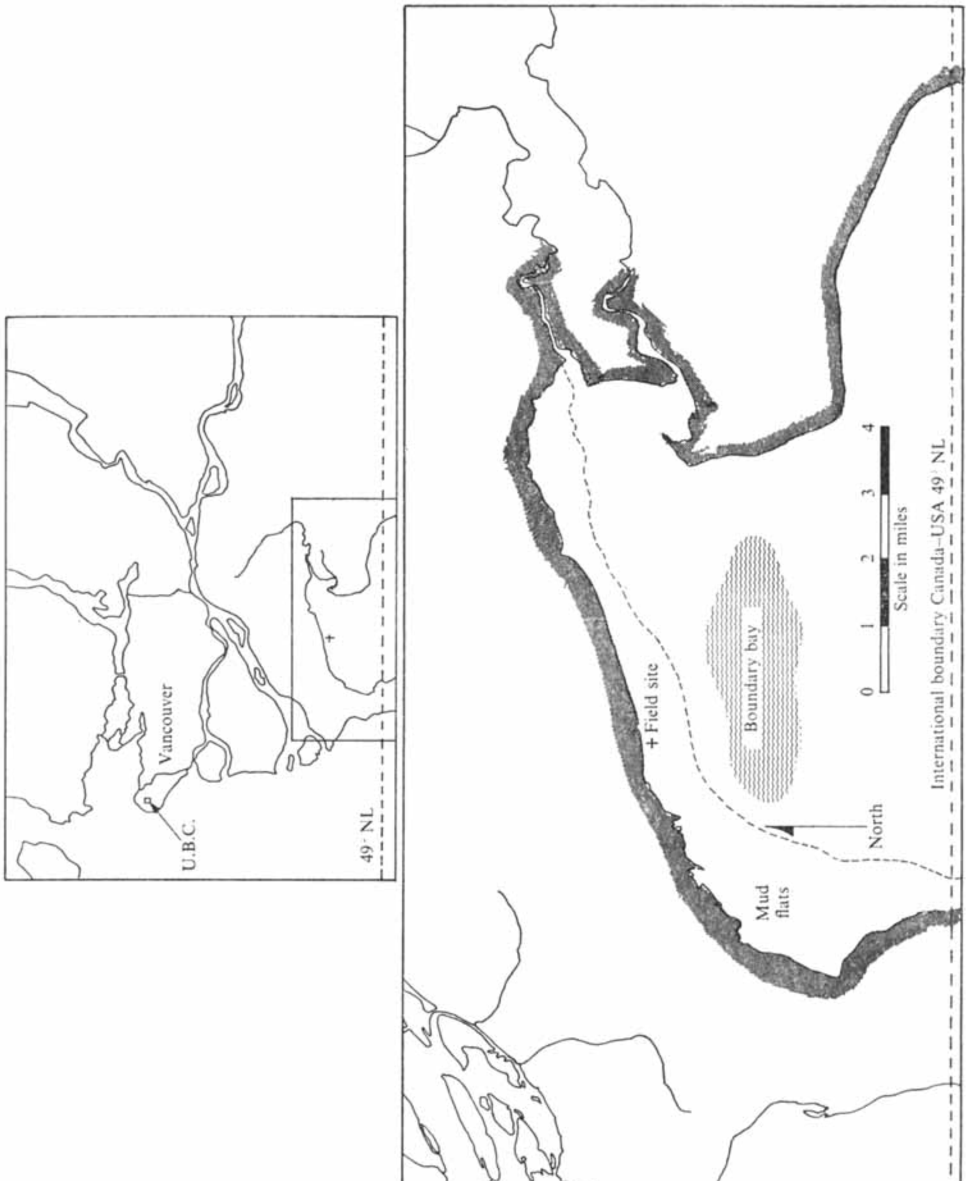
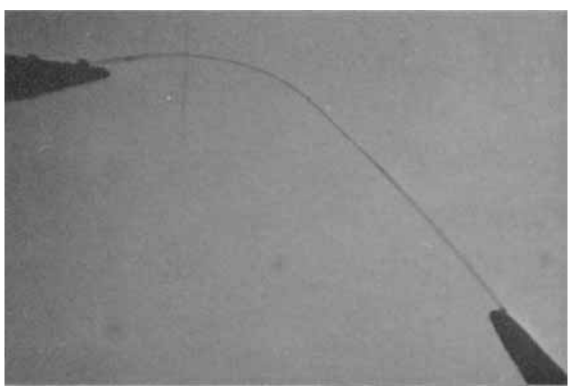


FIGURE 1. Location of field site.

(*a*)



(*b*)

FIGURE 2. Microscope photographs of platinum core used as the temperature fluctuation sensor (by C. Woodhouse). (*a*) 0·25 μm platinum wire core extending from silver jacket soldered to prongs. (*b*) Magnification of (*a*) to show core section (distance between silver-coated ends approximately 0·4 mm).

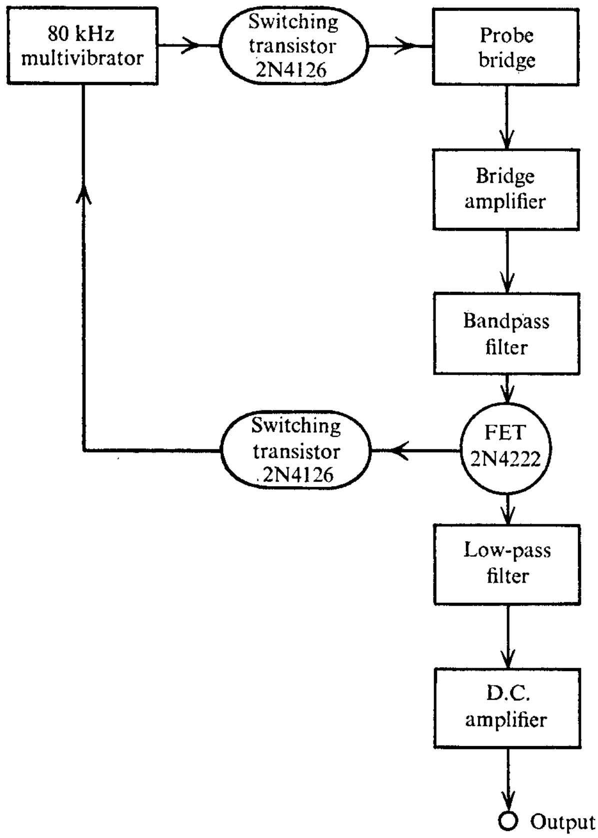


FIGURE 3. Block diagram of temperature fluctuation measurement system.

a current of 50 microamps. A change in wind speed of, for example, $4\,\mathrm{m\,s^{-1}}$ to $5\,\mathrm{m\,s^{-1}}$ would be registered as a temperature change of only 0·005 °C.

The signals from the platinum resistance thermometer, hot-wire and cup anemometer were recorded on separate f.m. (frequency modulated) channels of a magnetic tape recorder (Sangamo Model 3562) run at $60\,\mathrm{in.^{-1}\,s}$ (high frequency cut-off 20 kHz). The recording system is outlined in figure 4. The signal from the platinum resistance thermometer electronics following the low pass (l.p.) filter and the signal from the hot-wire anemometer electronics (55D05) were treated similarly. Each was recorded in two different ways chosen to optimize the incompatible requirements of noise performance and frequency response. These signals were recorded both unaltered (l.f. gain) and differentiated. The purpose of the differentiation circuit was to improve the signal-to-noise ratio at high frequencies. L.f. gain provided gain not only for the direct signal but also for the signal prior to differentiation. The high-frequency gain control (h.f. gain) determined the frequency at which there was unity gain in the differentiating circuit. H.f. gain was adjusted to provide optimum gain (prewhitening) of the frequencies of interest.

Data from five separate tapes were analysed. Sections were chosen from each of the tapes on the basis of stationarity, reasonably uniform levels of turbulence,

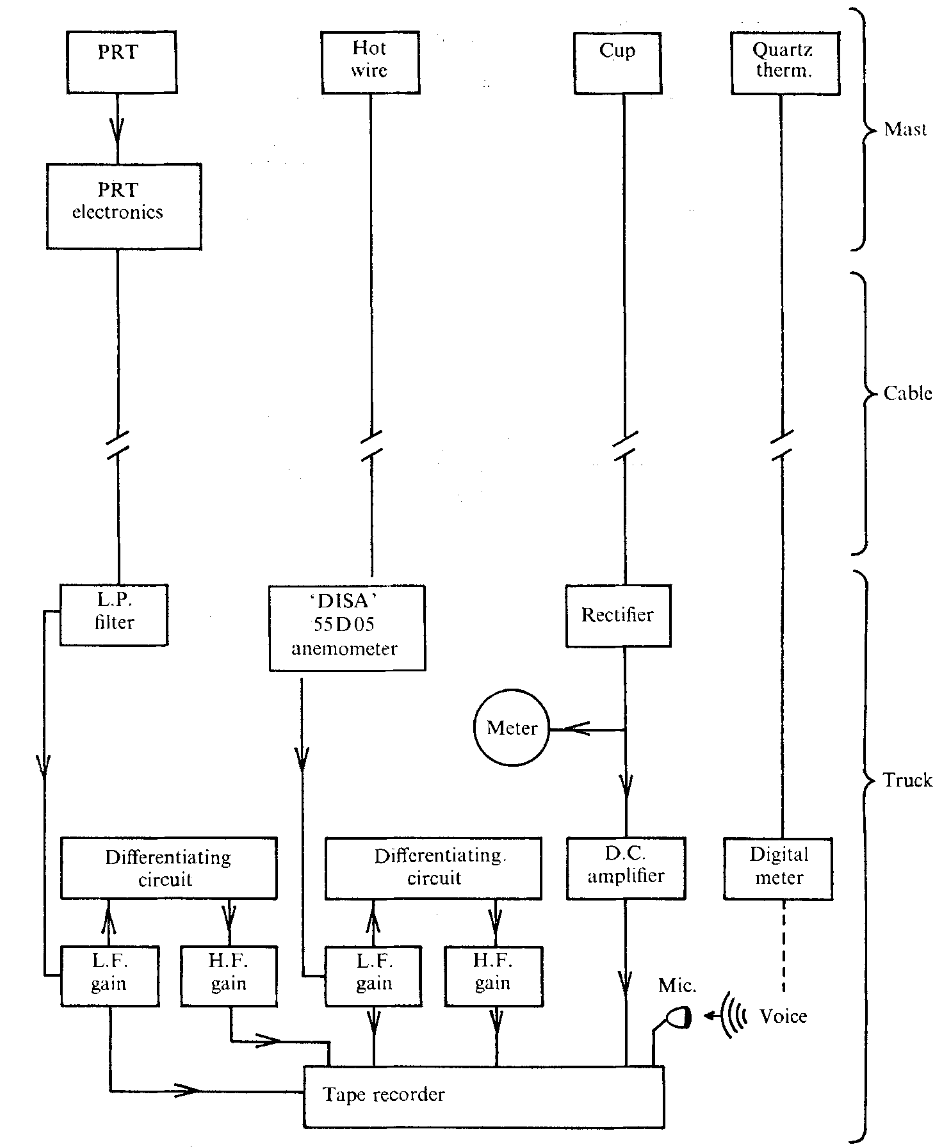


FIGURE 4. Block didagram of recording system (including a platinum resistance thermometer (PRT)).

mean winds and temperature. Section details are given in table 1. These data were collected on 8 August 1969 between noon and 8.00 p.m. Pacific Daylight time. The first run made appears at the top of table 1 and the last at the bottom. Nineteen cases of velocity spectra and sixteen cases of temperature spectra were examined.

The velocity derivative signal, velocity signal and temperature signal were digitized at a sampling rate of 2 kHz after l.p. filtering (filter cut-off 48 db octave^{-1}) at 1 kHz. The temperature derivative signal was digitized at 6 kHz after l.p. filtering at 2 kHz. The digitized data were processed on an IBM 360/67 computer

Tape no.	Record start time PDT†	Section	Mean wind (m/s)	Reynolds number‡ $Re \times 10^6$	Length of record (s)
202 (2)	1330	A	3·43	0·9	35
		B^2	3·39	0·9	30
—		D	3·50	1·0	60
		F	3·42	0·9	23
203 (1)	1427	A	3·35	0·9	60
		B	3·08	0·9	60
		C	3·00	0·8	60
		D	2·90	0·8	60
209 (3)	1600	A	3·86	1·1	60
		B	4·03	1·1	60
		C	3·70	1·0	60
		D	3·68	1·0	60
RCA 1(1)	1629	A	4·30	1·2	60
		B	4·20	1·2	60
		E	3·50	1·0	60
		F	2·90	0·8	60
RCA 3(1)	1945	A	3·65	1·0	60
		B	3·93	1·1	60
		C	4·50	1·2	60

$Z = 4$ m, $\nu =$ kinematic viscosity.
† Pacific Daylight Time.
‡ Reynolds number $= Re = UZ/\nu$.

TABLE 1. Listing of sections analysed

at the University of British Columbia. Typical spectra showing observed 80% confidence levels as vertical bars are shown in figure 5. The horizontal bars indicate the bandwidth over which the spectral value was estimated. Frequency spectra were converted to wavenumber spectra through direct application of Taylor's hypothesis ($k = 2\pi f/U$).

4. Results

4.1. *Velocity*

Velocity spectra were normalized according to (5) and the five groups (figure 6) compared with previous results. The 'universal' curve for velocity spectra determined by Grant *et al.* (1962) from oceanic turbulence is found to fit very closely to these groups of spectra. The group 202 (2) falls below the Grant *et al.* curve at high wavenumbers. Nasmyth (1970) has produced a new approximation to Kolmogorov's universal function based on very 'clean' spectra of oceanic turbulence. The new curve lies very near the old in the low and mid-range of wavenumbers, has a slightly sharper 'knee' and then falls below the old curve. His spectra were compared with the 202 (2) spectra and the Grant *et al.* spectra (figure 7). Since no 'noise' was subtracted from the 202 (2) spectra (or any of the other velocity spectra) and the signal-to-noise ratio was the best of the data

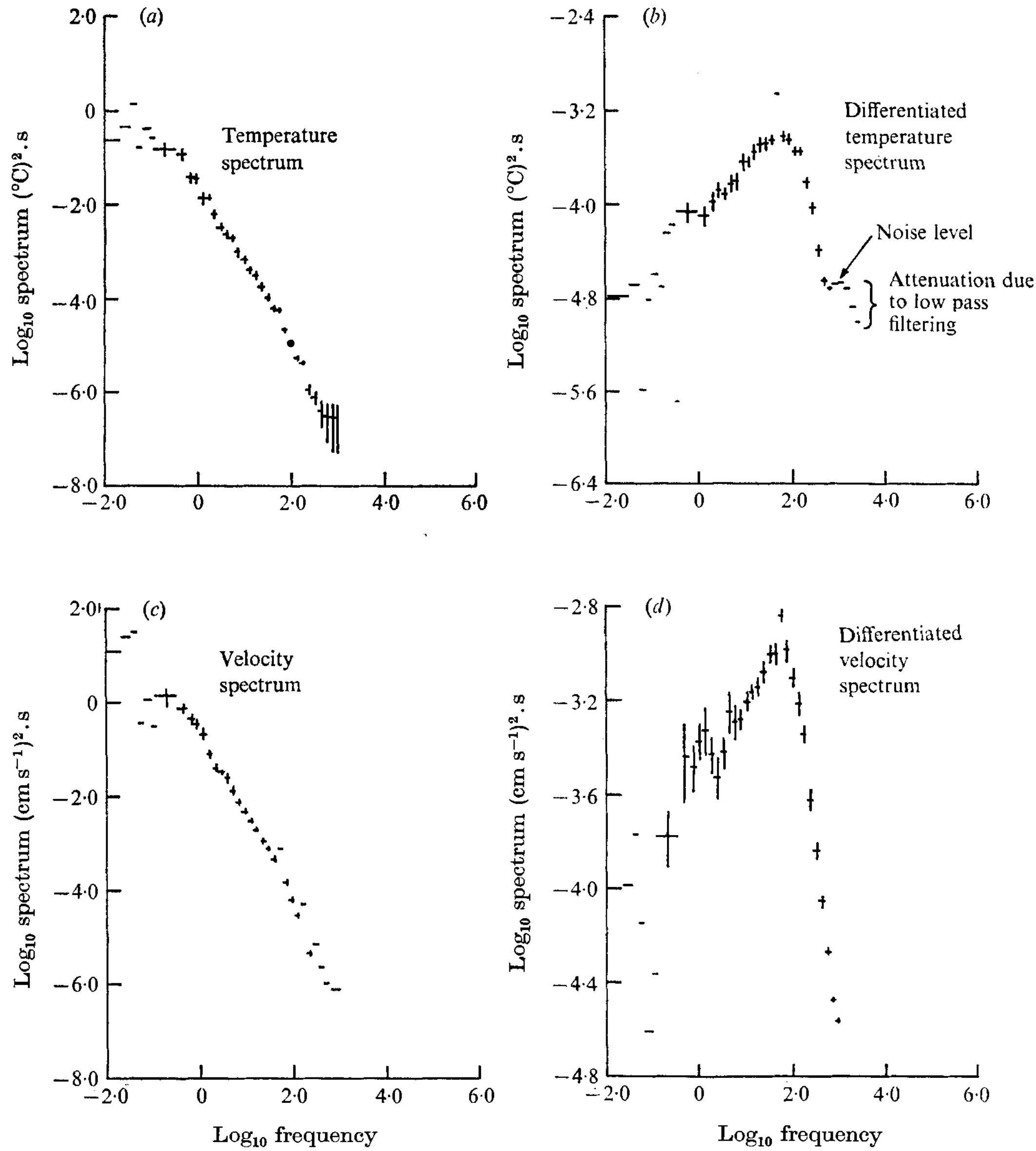


FIGURE 5. (a)–(d). Typical spectra illustrating 80 % confidence limits (vertical bars) and bandwidths (horizontal bars).

presented here, agreement over the viscous dissipation region can be taken as partial support for the universal curve proposed by Nasmyth. Those spectra that were not as 'clean' agree better with the old curve of Grant *et al.* (1962).

Energy dissipation spectra were normalized according to (6) and plotted in linear form (figure 8). These results compare very well with those of Pond (1965) and Grant *et al.* (1962). Dissipation spectrum 202 (2) A was compared with the dissipation spectrum presented by Gibson *et al.* (1970). The level of 202 (2) A is lower in the 'inertial' range but does not fall as quickly in the dissipation region.

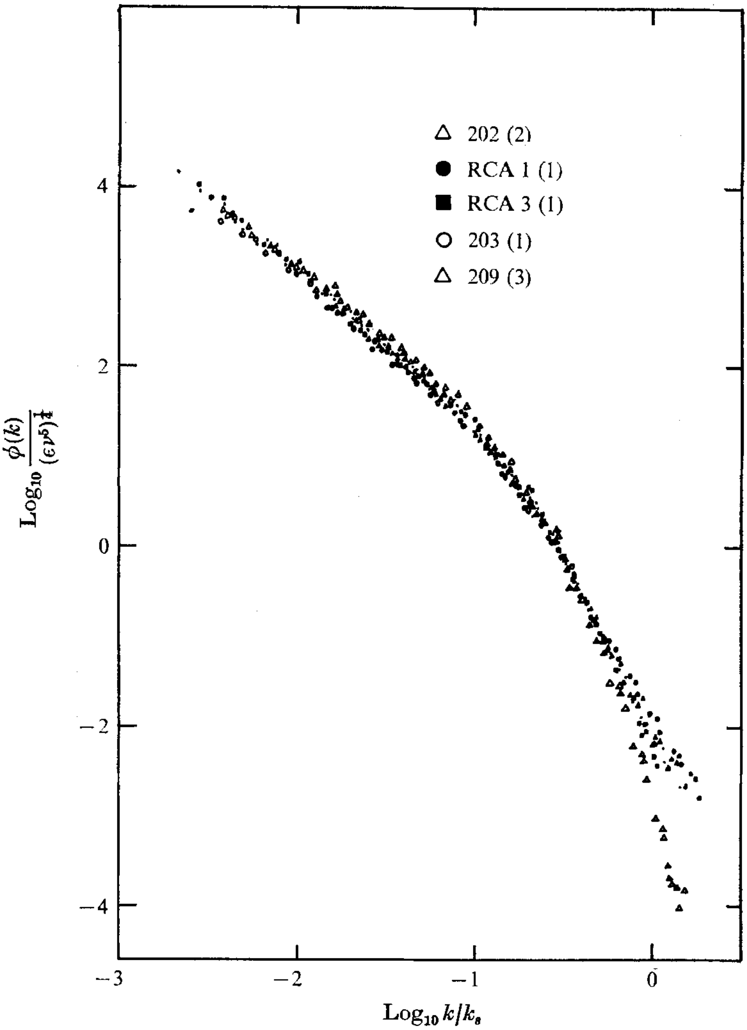


FIGURE 6. Normalized velocity spectra.

Since these data agree well with the Nasmyth 'universal' curve, then the Gibson *et al.* curve appears to fall at high wavenumbers faster than the present authors would expect.

The value of K' was estimated from (7) and the one-dimensional integral in (4). Based on 17 estimates (table 2) a value of 0·51 with a standard error of the mean of 0·02 was obtained. This agrees well with most previous values (§ 2.1), and it is not far from the recent values of 0·56 obtained by Nasmyth (1970) and 0·53 obtained by Stewart *et al.* (1970).

4.2. *Temperature*

Temperature spectra were normalized according to (13). These spectra (figure 9) match very well in the dissipation region where the normalization procedure requires the closest fit. This lends some confidence in the techniques used to

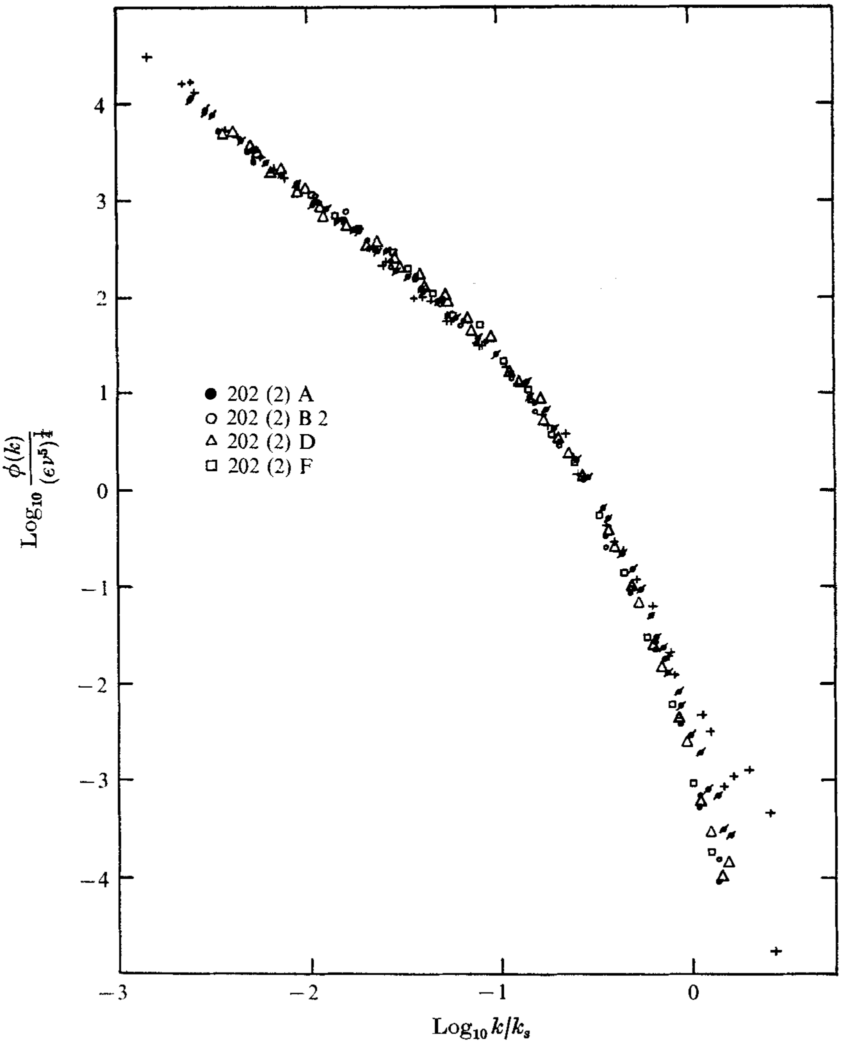


FIGURE 7. Comparison of 202 (2) normalized velocity spectra with (●) Nasmyth (1970) spectra and (+) Grant *et al.* (1962) spectra.

acquire and analyse these data. The spectra demonstrate clearly the shape of the one-dimensional temperature spectrum in air beyond the '$-\frac{5}{3}$' region. There is no '-1' region, which suggests that large σ theory does not apply to temperature fluctuations in air.

The universal temperature and velocity spectra are very similar (figure 10). The temperature spectra tends to be slightly higher than, and displaced slightly to the right of, the velocity spectra. The curves defining the 'dissipation' regions appear to parallel each other.

There are few previous measurements with which to compare these data. The results of, for example, Gibson & Schwarz (1963) compare favourably in the '$-\frac{5}{3}$' region but the agreement departs at high wavenumber due to Prandtl number effects. The Prandtl numbers of the fluids they worked with were 7 and

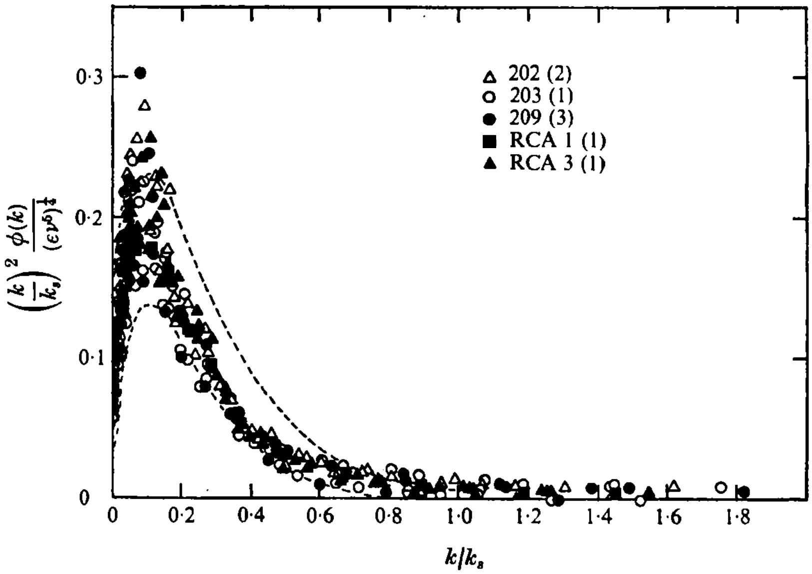


FIGURE 8. Normalized energy dissipation spectra. Dashed line is envelope of points from Grant *et al.* (1962).

Tape no.	Section	ϵ cm^2 s^{-3}	ϵ_θ (°C)^2 s^{-1}	K'	K'_θ
202 (2)	A	64	0·12	—	0·83
	B2	69	0·058	0·50	0·75
	D	46	0·049	0·42	0·70
	F	103	0·036	0·47	0·90
203 (1)	A	45	0·043	0·41	0·72
	B	44	0·047	0·40	0·72
	C	82	0·048	0·41	0·82
	D	48	0·020	0·37	0·72
209 (3)	A	62	0·023	0·48	0·79
	B	33	0·049	0·60	0·77
	C	91	0·033	—	0·85
	D	57	0·091	0·59	0·83
RCA 1 (1)	A	63	0·018	0·48	0·86
	B	95	0·014	0·51	0·86
	E	100	0·013	0·56	0·99
	F	125	0·016	0·61	0·90
RCA 3 (1)	A	184	—	0·53	—
	B	130	—	0·62	—
	C	135	—	0·66	—
			Average	0·51	0·81
			Standard error of mean	0·02	0·02
			Standard deviation	0·09	

TABLE 2. 'Inertial range' constants

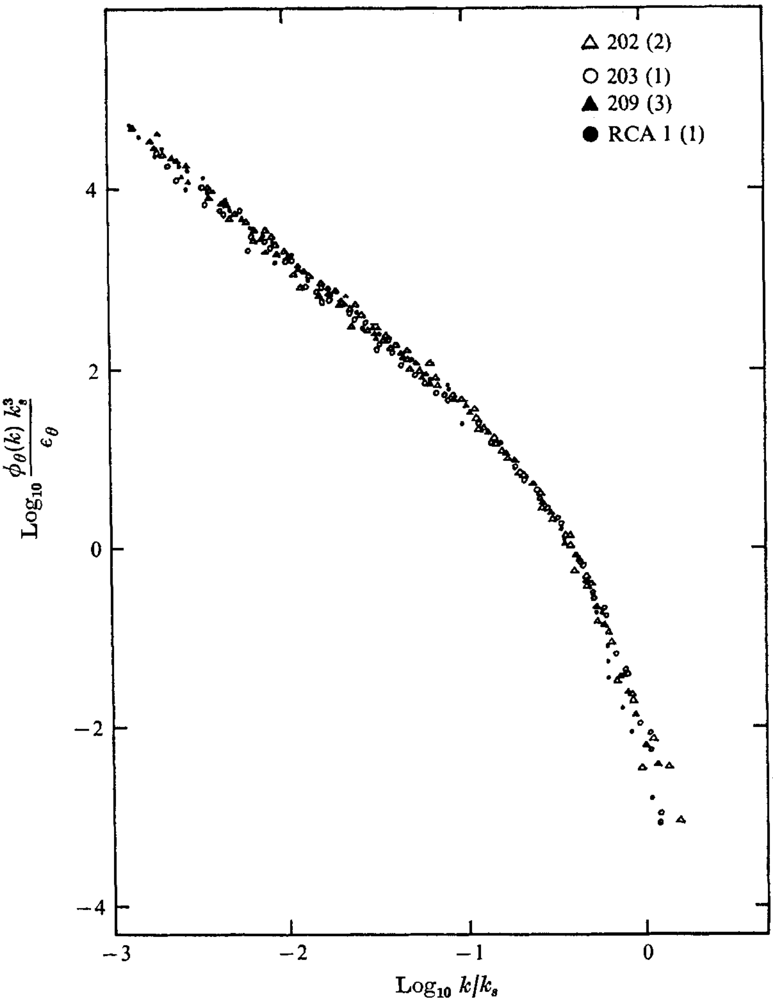


FIGURE 9. Normalized temperature spectra.

700. On the other hand, Lanza & Schwarz (1966), who as in this study measured temperature fluctuations in air, produced universal spectra which agreed at high wavenumbers but not at low wavenumbers.

Temperature dissipation spectra were normalized according to (14) and plotted linearly (figure 11). The points lie within a moderately narrow envelope. Temperature dissipation spectrum 203 (1)D was compared with the temperature dissipation spectrum presented by Gibson *et al.* (1970). The log–log plot (figure 12) shows discrepancies in the two spectra which are similar to the discrepancies noted in the velocity dissipation spectra of these results and of Gibson *et al.* Even though a 'noise' subtraction was made from 203 (1)D (and the rest of the temperature dissipation spectra) the fall-off rate does not approach that of the Gibson *et al.* spectrum. Gibson (private communication) is of the opinion that the rapid fall-off in their spectrum is due to the 36 db octave^{-1} filter cut-off used in the analysis of the data.

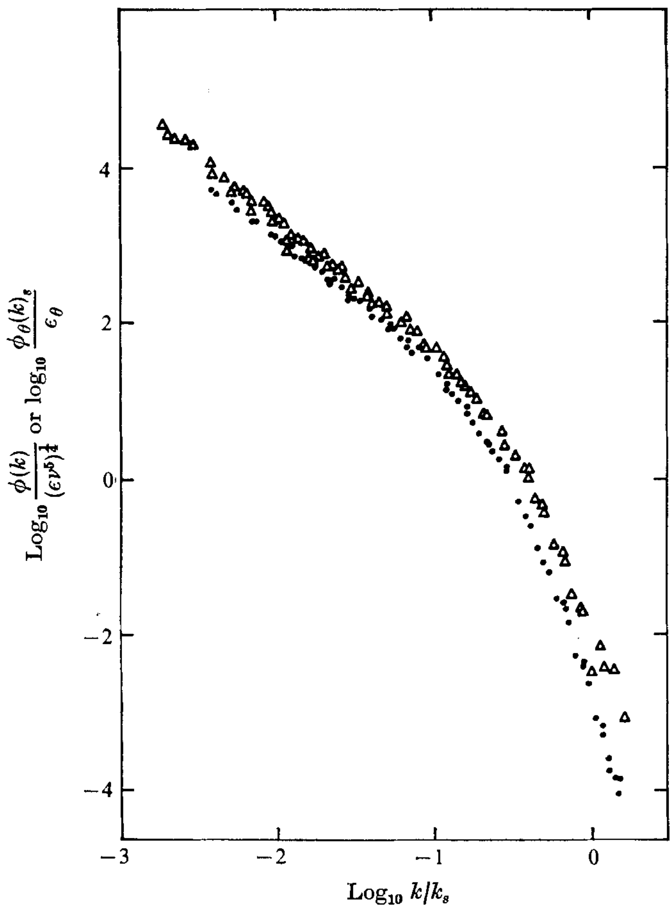


FIGURE 10. Comparison of normalized 202 (2) temperature (△) and velocity (●) spectra.

The scalar constant K'_θ was estimated from (15) and the one-dimensional integral in (12). Based on 16 estimates (table 2), a value of 0·81 with a standard error of the mean of 0·02 was obtained. The previous scatter has been commented on (§ 2.2). The previous values for the most part have been estimated by indirect though none the less valid methods. This evaluation of K'_θ has been made by direct measurement of all quantities which enter into the calculation of it.

5. Summary

These results support previous results of velocity spectra but suggest that the universal curve may be slightly high at large wavenumbers. The value of the one-dimensional constant is only slightly higher than formerly believed in 1966.

The shape of the one-dimensional temperature spectrum in air beyond the $-\frac{5}{3}$ region has been clearly illustrated and does not show a '-1' region. A Prandtl number of 0·7 is not a large Prandtl number which means that large Prandtl number theory is not applicable to temperature fluctuations in air. The value of

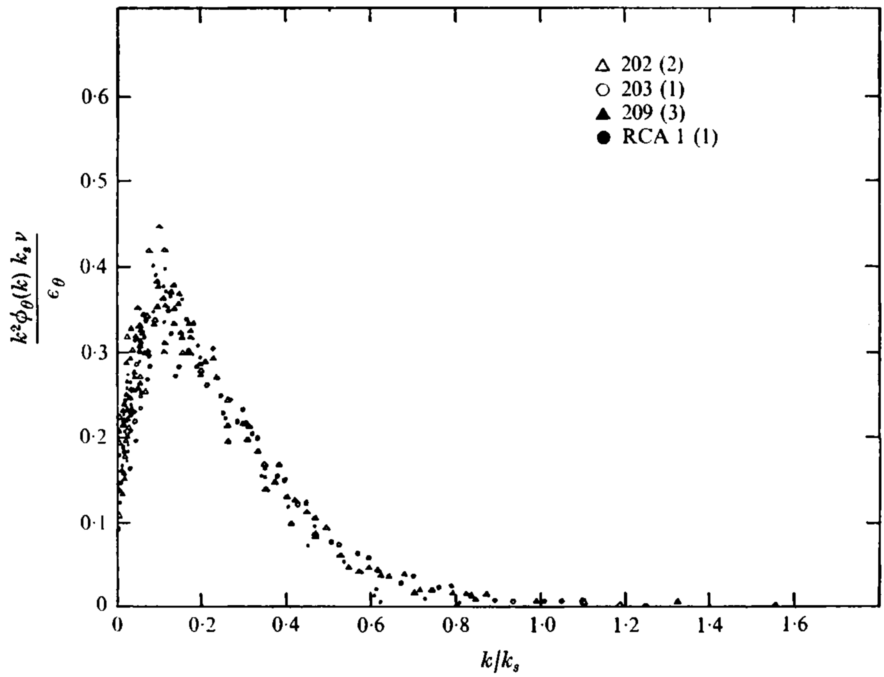


FIGURE 11. Normalized temperature (scalar) dissipation spectra.

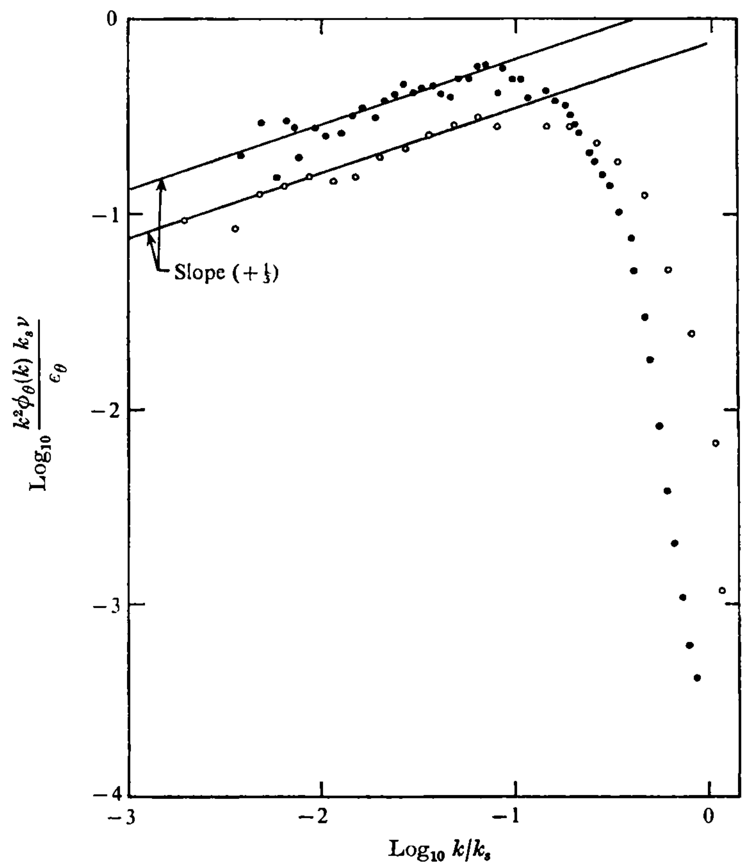


FIGURE 12. Comparison of (○) 203 (1)D and (●) Gibson *et al.* (1970) differentiated temperature (scalar) dissipation spectra.

the scalar one-dimensional constant is about 0·8. A clear and concise discussion of various values of the scalar constant has been given by Paquin & Pond (1971). Clearly a problem exists, and further careful experimental work is needed.

This work was supported under DRB grant 9550–09 for 'Low Speed Fluid Dynamics'. This work is a part of more general programmes supported by NRC (Canada), Meteorological Branch of D.O.T. (Canada) and the Naval Ordnance Systems Command, ONR (U.S.A.). We wish to thank Dr R. W. Stewart, who was helpful throughout the course of this work, and Dr Carl Gibson, who made a number of useful suggestions for improving the original manuscript. We have also incorporated a number of suggestions by two anonymous reviewers, who did a splendid job of editing.

REFERENCES

Batchelor, G. K. 1953 *The Theory of Homogeneous Turbulence*. Cambridge University Press.

Batchelor, G. K. 1959 Small-scale variation of convected quantities like temperature in a turbulent fluid. Part 1. *J. Fluid Mech.* **5**, 113.

Batchelor, G. K., Howells, J. D. & Townsend, A. A. 1959 Small-scale variation of convected quantities like temperature in a turbulent fluid. Part 2. *J. Fluid Mech.* **5**, 134.

Boston, N. E. J. 1970 An investigation of high-wavenumber temperature and velocity spectra in air. Ph.D. thesis, University of British Columbia.

Corrsin, S. 1951 On the spectrum of isotropic temperature fluctuations in an isotropic turbulence. *J. Appl. Phys.* **22**, 469.

Gibson, C. H. 1962 Scalar mixing in turbulent flow. Ph.D. thesis, Stanford University

Gibson, C. H. 1968*a* Fine structure of scalar fields mixed by turbulence. 1. Zero-gradient points and minimal gradient surfaces. *Phys. Fluids*, **11**, 2305.

Gibson, C. H. 1968*b* Fine structure of scalar fields mixed by turbulence. 2. Spectral theory. *Phys. Fluids*, **11**, 2316.

Gibson, C. H. & Schwarz, W. H. 1963 The universal equilibrium spectra of turbulent velocity and scalar fields. *J. Fluid Mech.* **16**, 365.

Gibson, C. H., Stegen, G. R. & Williams, R. B. 1970 Statistics of the fine structure of turbulent and temperature fields measured at high Reynolds number. *J. Fluid Mech.* **41**, 153.

Gibson, M. M. 1963 Spectra of turbulence in a round jet. *J. Fluid Mech.* **15**, 161.

Granatstein, V. L., Buchsbaum, S. J. & Bugnolo, D. S. 1966 Fluctuation spectrum of a plasma additive in a turbulent gas. *Phys. Rev. Lett.* **16**, 504.

Grant, H. L., Hughes, B. A., Vogel, W. M. & Moilliet, A. 1968 The spectrum of temperature fluctuations in turbulent flow. *J. Fluid Mech.* **34**, 423.

Grant, H. L., Stewart, R. W. & Moilliet, A. 1962 Turbulence spectra from a tidal channel. *J. Fluid Mech.* **12**, 241.

Gurvich, A. S. & Kravchenko, T. L. 1962 On the frequency spectrum of temperature fluctuations in the small-scale region. *Trudy Inst. Atmospheric Phys. ANSSSR*, **4**, 144.

Gurvich, A. S. & Zubkovski, S. L. 1966 Evaluation of structural characteristics of temperature pulses in the atmosphere. *Izv. Acad. Sci., USSR, Atmos. Oceanic Phys.* **2** (2), 118–120.

Hinze, J. O. 1959 *Turbulence*. McGraw-Hill.

Howells, I. D. 1960 An approximate equation for the spectrum of a conserved scalar quantity in a turbulent fluid. *J. Fluid Mech.* **9**, 104.

Kistler, A. L. & Vrebalovich, T. 1966 Grid turbulence at large Reynolds numbers. *J. Fluid Mech.* **26**, 37.

Kolmogorov, A. N. 1941 The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. *C.R. Acad. Sci. U.R.S.S.* **30**, 301.

Kolmogorov, A. N. 1962 A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number. *J. Fluid Mech.* **13**, 82.

Kraichnan, R. H. 1968 Small-scale structure of a scalar field convected by turbulence. *Phys. Fluids*, **11**, 945.

Lanza, J. & Schwarz, W. H. 1966 The scalar spectrum for the equilibrium range of wavenumbers. Department of Chemical Engineering, Stanford University.

Lumley, J. L. & Panofsky, H. A. 1964 *The Structure of Atmospheric Turbulence*. Wiley.

Nasmyth, P. W. 1970 Oceanic turbulence. Ph.D. thesis, University of British Columbia.

Obukhov, A. M. 1949 Structure of the temperature field in turbulent streams. *Izvestia ANSSSR, Geogr. Geophys.* no. 13, p. 58.

Pao, Y. 1965 Structure of turbulent velocity and scalar fields at large wavenumbers. *Phys. Fluids*, **8**, 1063.

Paquin, J. E. & Pond, S. 1971 The determination of the Kolmogoroff constants for velocity, temperature and humidity fluctuations from second- and third-order structure functions. *J. Fluid Mech.* **50**, 257.

Pond, S. 1965 Turbulence spectra in the atmospheric boundary layer over the sea. Ph.D. thesis, University of British Columbia.

Pond, S., Smith, S. D., Hamblin, P. F. & Burling, R. W. 1966 Spectra of velocity and temperature fluctuations in the atmospheric boundary layer over the sea. *J. Atmos. Sci.* **23**, 376.

Pond, S., Stewart, R. W. & Burling, R. W. 1963 Turbulence spectra in the wind over waves. *J. Atmos. Sci.* **20**, 319.

Rust, H. H. & Sesonke, A. 1966 Turbulent temperature fluctuations in mercury and ethylene glycol in pipe flow. *Int. J. Heat Mass Transfer*, **9**, 215.

Sheih, C. M., Tennekes, H. & Lumley, J. L. 1971 Airborne hot-wire measurements of the small-scale structure of atmospheric turbulence. *Phys. Fluids*, **14**, 201.

Stewart, R. W. & Townsend, A. A. 1951 Similarity and self-preservation in isotropic turbulence. *Phil. Trans.* A **243**, 359.

Stewart, R. W., Wilson, J. R. & Burling, R. W. 1970 Some statistical properties of small-scale turbulence in an atmospheric boundary layer. *J. Fluid Mech.* **41**, 141.

Taylor, G. I. 1938 The spectrum of turbulence. *Proc. Roy. Soc.* A **164**, 476.

Van Atta, C. W. 1971 Influence of fluctuations in local dissipation rates on turbulent scalar characteristics in the inertial subrange. *Phys. Fluids*, **14**, 1803.

Weiler, H. S. & Burling, R. W. 1967 Direct measurements of stress and spectra of turbulence in the boundary layer over the sea. *J. Atmos. Sci.* **24**, 653.

Wyngaard, J. C. & Tennekes, H. 1970 Measurements of the small-scale structure of turbulence at moderate Reynolds numbers. *Phys. Fluids*, **13**, 1962.

Yaglom, A. M. 1966 *Sov. Phys. Dokl.* **11**, 26